Q.1

$$\sum_{n=a}^{b} f(n) = f(a)+f(a+1)+f(a+2)+\dots+f(b)$$

$a, b \in \mathbb{Z}$

BEWARE: $$\sum_{n=3}^{5} i^2 = i^2+i^2+i^2 = 3i^2$$

LINEAR PROPERTIES OF SUMMATION

[RECALL $\sum_{i=1}^{n} a_i = a_1+a_2+a_3+\dots+a_n$]

$$\sum_{i=1}^{n}(a_i \pm b_i) = (a_1 \pm b_1)+(a_2 \pm b_2)+(a_3 \pm b_3)+\dots+(a_n \pm b_n)$$

$$= (a_1+a_2+a_3+\dots+a_n) \pm (b_1+b_2+b_3+\dots+b_n)$$

① $$\sum_{i=1}^{n}(a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

$$\sum_{i=1}^{n} ca_i = ca_1+ca_2+ca_3+\dots+ca_n = c(a_1+a_2+a_3+\dots+a_n)$$

$(c \in \mathbb{R})$

② $$\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$$

$$\sum_{i=1}^{n} c = \underbrace{c+c+c+\cdots+c}_{n \text{ TERMS}} = cn$$

$$\text{eg. } \overset{n \rightarrow}{\sum_{i=1}^{54}} \underset{\overset{\uparrow}{c}}{2} = \underset{\overset{\uparrow}{cn}}{2(54)} = 108$$

$$\sum_{i=1}^{n} i = 1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

$$\text{eg. } \overset{n \rightarrow}{\sum_{i=1}^{5}} i = \frac{5(\cancel{6})^{3}}{\cancel{2}} = 15$$

CHECK: $1+2+3+4+5 = 15$

$$\sum_{i=1}^{1000} i = \frac{\overset{500}{\cancel{1000}}(1001)}{\cancel{2}} = 500500$$

$$\sum_{i=1}^{20}(5i+2) = \sum_{i=1}^{20} 5i + \sum_{i=1}^{20} 2 = 5\sum_{i=1}^{20} i + \sum_{i=1}^{20} 2 = 5 \cdot \frac{\overset{10}{\cancel{20}}(21)}{\cancel{2}} + 2(20)$$

$$= 1050 + 40$$

$$= 1090$$

$\downarrow$

$(5(1)+2)+(5(2)+2)+(5(3)+2)+\cdots+(5(20)+2)$

$$\sum_{i=1}^{n} i^2 = 1^2+2^2+3^2+\cdots+n^2$$

$$= 1+4+9+\cdots+n^2$$

$$= \frac{n(n+1)(2n+1)}{6}$$

$$\text{CHECK: } \overset{n \rightarrow}{\sum_{i=1}^{4}} i^2 = \frac{\overset{2}{\cancel{4}}(5)\overset{3}{\cancel{(9)}}}{\cancel{6}} = 30$$

$$1^2+2^2+3^2+4^2 = \overbrace{1+\underbrace{4+9+16}_{20}}^{10} = 30$$

9.2 ARITHMETIC SEQUENCES + SERIES

eg. $2, 5, 8, 11, 14, 17, 20$

$+3 \quad +3 \quad +3 \quad +3 \quad +3 \quad +3$

DEFINITION: AN ARITHMETIC SEQUENCE IS A SEQUENCE IN WHICH EACH TERM (EXCEPT THE FIRST) IS THE PREVIOUS TERM PLUS A FIXED CONSTANT (CALLED THE COMMON DIFFERENCE)

ARE THESE ARITHMETIC SEQUENCES?

① $1^2, 2^2, 3^2, 4^2, 5^2, 6^2$

$1, 4, 9, 16, 25, 36$ NO

$4-1=$③ $9-4=$⑤

② $\frac{1}{4}, \frac{1}{3}, \frac{5}{12}, \frac{1}{2}, \frac{7}{12}, \frac{2}{3}, \frac{3}{4}$ YES $\rightarrow$ $\frac{3}{12}, \frac{4}{12}, \frac{5}{12}, \frac{6}{12}, \frac{7}{12}, \frac{8}{12}, \frac{9}{12}$

$+\frac{1}{12}$ " " " " "

$\frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12}$

CHECK: $\frac{1}{3} + \frac{1}{12} = \frac{4+1}{12} = \frac{5}{12}$ ✓

$\frac{5}{12} + \frac{1}{12} = \frac{6}{12} = \frac{1}{2}$ ✓

$\frac{6}{12} + \frac{1}{12} = \frac{7}{12}$ ✓

$\frac{7}{12} + \frac{1}{12} = \frac{8}{12} = \frac{2}{3}$ ✓

$\frac{8}{12} + \frac{1}{12} = \frac{9}{12} = \frac{3}{4}$ ✓

③ $\ln 2, \ln 4, \ln 8, \ln 16, \ln 32$ YES

$\ln A - \ln B = \ln \frac{A}{B}$

$\ln A + \ln B = \ln(AB)$

$\ln A^B = B \ln A$

$\ln 4 - \ln 2 = \ln \frac{4}{2} = \ln 2$

CHECK: $\ln 4 + \ln 2 = \ln(4 \cdot 2) = \ln 8$ ✓

$\ln 8 + \ln 2 = \ln(8 \cdot 2) = \ln 16$ ✓

$\ln 16 + \ln 2 = \ln(16 \cdot 2) = \ln 32$ ✓

$\ln 2, \ln 2^2, \ln 2^3, \ln 2^4, \ln 2^5$

$\ln 2, 2\ln 2, 3\ln 2, 4\ln 2, 5\ln 2$

$+\ln 2$ " " "

GENERAL FORMULA FOR ARITHMETIC SEQUENCE
WITH FIRST TERM a_1 AND COMMON DIFFERENCE d

$a_1 = a_1 \qquad = a_1 + 0d$

$a_2 = a_1 + d \qquad = a_1 + 1d$

$a_3 = a_2 + d = (a_1 + d) + d = a_1 + 2d$

$a_4 = a_3 + d = (a_1 + 2d) + d = a_1 + 3d$

$a_n = \quad \sim\sim\sim \quad = a_1 + (n-1)d$

$$a_n = a_1 + (n-1)d$$

FIND THE G.F. FOR $\frac{1}{4}, \frac{1}{3}, \frac{5}{12}, \frac{1}{2}, \frac{7}{12}, \frac{2}{3}, \frac{3}{4}$

$d = +\frac{1}{12}$

$a_1 = \frac{1}{4}$

$$a_n = a_1 + (n-1)d$$

$$a_n = \frac{1}{4} + (n-1)\frac{1}{12}$$

$$a_n = \frac{1}{4} + \frac{1}{12}(n-1)$$

CHECK: $a_6 = \frac{1}{4} + \frac{1}{12}(5)$

$$= \frac{3+5}{12}$$

$$= \frac{8}{12} = \frac{2}{3} \checkmark$$

FIND THE 51^{ST} TERM OF THE SEQUENCE $25, 21, 17, 13, 9, \ldots$

$+(-4) \quad +(-4) \quad +(-4) \quad +(-4)$

$21 - 25 = -4$

$$a_n = a_1 + (n-1)d$$

$$= 25 - 4(n-1)$$

$$a_{51} = 25 - 4(51-1)$$

$$= 25 - 4(50)$$

$$= 25 - 200$$

$$= -175$$